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SUSY and Goliath

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Abstract: We investigate the "giant gravitons" of McGreevy, Susskind and Toumbas [1]. We demonstrate that these are BPS configurations which preserve precisely the same supersymmetries as a "point-like" graviton. We also show that there exist "dual" giant gravitons consisting of spherical branes expanding into the AdS component of the spacetime. Finally, we discuss the realization of the stringy exclusion principle within this expanded framework.

Keywords: M-Theory, p-branes, D-branes, AdS-CFT Correspondance.

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Contents

1. Introduction

One of the intriguing observations to arise in the AdS/CFT correspondence [2] — see also the review [3] — is the "stringy exclusion principle" [4]. In the conformal field theory, this result is easily understood. A family of chiral primary operators in the superconformal field theory terminates at some maximum weight because the gauge symmetry group has a finite rank. In terms of the dual anti-de Sitter description, these operators are associated with single particle states carrying angular momentum on the internal (spherical) geometry. The appearance of an upper bound on the angular momentum seems mysterious from the point of view of the supergravity theory. Recently, however, McGreevy, Susskind and Toumbas [1] provided an ingenious mechanism for how the upper bound appears. Rather surprisingly, their resolution arises through a large distance phenomenon. They suggested that the supergraviton states expand into the spherical part of the space-time geometry with a radius proportional to the angular momentum. The radius of these "giant gravitons" must be smaller than the radius of the sphere, which through the AdS/CFT is related to the rank of the gauge group in the CFT. Thus they are able to reproduce precisely the desired upper bound on the angular momentum.

As a consistency check on this picture, it is important that the energy of this giant graviton state equals the energy of the usual graviton [1]. This also suggests that the giant graviton is a BPS state, preserving some of the supersymmetry. On the other hand, one might think that these spherical brane configurations would break all the supersymmetries. Antipodal patches of the sphere can be regarded as parallel portions of branes and anti-branes, so preserving any supersymmetry is highly nontrivial. We will analyze the supersymmetry properties of these expanded branes, and we show that they indeed are supersymmetric, and further that the giant graviton preserves precisely the same supersymmetries as a "point-like" graviton.

This supersymmetry analysis thus strengthens the giant graviton picture, but we will also point out a potential problem with the interpretation in terms of the stringy exclusion principle. The problem is the existence of other configurations that could also play the role of the graviton. In particular, we will show that there are "dual" giant gravitons, that is, configurations in which a dual brane expands into the AdS part of the spacetime. This brane configuration *also* has the right energy and supersymmetries to represent the graviton carrying a fixed angular momentum. Its radius is again proportional to the angular momentum on the spherical part of the spacetime, but because the expansion now occurs in the non-compact AdS space, there is no upper bound implied.

Hence, instead of a unique candidate for the graviton state, we have at least *three* different ones, including the point-like gravitons which already arise in the analysis of ref. [1]. Further, since two of the candidates display no upper bound on the angular momentum, there seems to be a problem for the proposed mechanism for the stringy exclusion principle. Quantum mechanically one might expect these different states to tunnel into each other, forming a unique ground state. In this direction, we show that there are finite-action instanton configurations describing tunneling between the expanded branes and their zero-size cousins. We suggest that the exclusion principle might still be realized within this expanded framework, by speculating that supersymmetry is spontaneously broken in the regime where there are only two possible graviton states, i.e. when the angular momentum bound is exceeded.

The paper is organized as follows. We start in section 2 by defining the various supergravity backgrounds and review the giant graviton construction in section 2.1. In section 2.2, we then describe the "dual" giant gravitons where the branes expand into the AdS space, and we follow with an analysis of the instanton transitions in section 2.3. Section 3 gives an analysis of which supersymmetries are preserved by the various brane configurations. We conclude in section 4 with a discussion of our results and the possible realization of the exclusion principle.

JHEP08(2000)040

As this paper was being completed, we were informed about work by Hashimoto, Hirano and Itzhaki [5] which has a significant overlap with the present paper.

2. Expanding branes

We wish to consider various (test) brane configurations in background spacetimes of the form $AdS_m \times S^n$. We will be considering M2- and M5-branes in the $D = 11$ supergravity backgrounds with $(m, n) = (4,7)$ and $(7,4)$, and also D3-branes in the type-IIb supergravity background with $(m, n) = (5,5)$. We will give a general presentation of the brane solutions which encompasses all the three cases simultaneously.

The full line element for the metric on $AdS_m \times S^n$ takes the form $ds^2 = ds^2_{\rm AdS} + ds^2_{\rm sph}$. We will use global coordinates on AdS_m, with

$$ds^2_{\rm AdS} = -\left(1+\frac{r^2}{\widetilde{L}^2}\right)dt^2 + \frac{dr^2}{1+\frac{r^2}{\widetilde{L}^2}} + r^2 d\Omega^2_{m-2}\,, \tag{2.1}$$

and our coordinates on the sphere will be

$$ds^2_{\rm sph} = L^2\left(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\Omega^2_{n-2}\right). \tag{2.2}$$

The radius of curvature for the AdS_m is $\widetilde{L}$, while that for the S^n is L. For the three cases in which we are interested,

$$(m,n) = \begin{cases} (4,7)\,, & L = 2\widetilde{L}\,; \\ (5,5)\,, & L = \widetilde{L}\,; \\ (7,4)\,, & L = \widetilde{L}/2\,; \end{cases} \tag{2.3}$$

or simply $L = \frac{n-3}{2}\widetilde{L}$ (see, for example, refs. [6, 3]). For the following calculations, a convenient explicit choice of coordinates for the spherical component of the AdS metric (2.1) is

$$d\Omega^2_{m-2} = d\alpha_1^2 + \sin^2\alpha_1\left(d\alpha_2^2 + \sin^2\alpha_2\left(\cdots + \sin^2\alpha_{m-3}d\alpha^2_{m-2}\right)\right), \tag{2.4}$$

and for the S^n metric (2.2),

$$d\Omega^2_{n-2} = d\chi_1^2 + \sin^2\chi_1\left(d\chi_2^2 + \sin^2\chi_2\left(\cdots + \sin^2\chi_{n-3}d\chi^2_{n-2}\right)\right). \tag{2.5}$$

These geometries along with the appropriate form gauge fields, for the three cases listed in eq. (2.3), comprise maximally supersymmetric solutions of the corresponding supergravity. The eleven-dimensional supergravity solutions are naturally given in terms of the four-form field strength, which is then proportional to the volume form on the four-dimensional component of the geometry [7]. Alternatively, these solutions can be written in terms of the Hodge-dual seven-form field strength, which is proportional to the volume form on the complementary seven-dimensional part of

JHEP08(2000)040

the space. For the type-IIb supergravity solution, the Freund-Rubin ansatz involves choosing the self-dual five-form field strength with terms proportional to the volume form on both of the five-dimensional factors of the full spacetime. The branes in these theories couple to the corresponding potentials. With the coordinates chosen above, we explicitly write the $(n-1)$-form potential on the S^n as

$$A^{(n-1)}_{\phi\chi_1\cdots\chi_{n-2}} = \beta_n L^{n-1} \sin^{n-1}\theta \, \sin^{n-3}\chi_1 \cdots \sin\chi_{n-3} \equiv \beta_n L^{n-1} \sin^{n-1}\theta \, \sqrt{g_\chi}\,, \qquad (2.6)$$

where $\sqrt{g_\chi}$ is the volume element on the unit $(n-2)$-sphere described by eq. (2.5). The constant β_n is simply a sign: $\beta_4 = +1 = \beta_5$, while $\beta_7 = -1$. These sign choices are made so that the four-form field strength appears with a positive coefficient in both of the M-theory backgrounds. Given these choices, the $(m-1)$-form potential on the AdS part of the space is written

$$A^{(m-1)}_{t\alpha_1\cdots\alpha_{m-2}} = -\frac{r^{m-1}}{\widetilde{L}} \sin^{m-3}\alpha_1 \cdots \sin\alpha_{m-3} \equiv -\frac{r^{m-1}}{\widetilde{L}} \sqrt{g_\alpha}\,, \qquad (2.7)$$

where $\sqrt{g_\alpha}$ is the volume element on the unit $(m-2)$-sphere described in eq. (2.4).

2.1 Giant gravitons revisited

McGreevy, Susskind and Toumbas [1] recently examined branes carrying angular momentum on the S^n, and they discovered unusual stable configurations in which an $(n-2)$-brane had expanded into the sphere part of the background geometry. In this section, we review these results.

For all of the cases of interest, the p-brane action may be written as

$$\begin{aligned} S_p &= -T_p \int d^{p+1}\sigma \sqrt{-g} + T_p \int P\left[A^{(p+1)}\right] \\ &= -T_p \int d^{p+1}\sigma \sqrt{-g} \left[1 + \frac{1}{(p+1)!}\, \varepsilon^{i_0\cdots i_p}\, \partial_{i_0} X^{M_0} \cdots \partial_{i_p} X^{M_p}\, A^{(p+1)}_{M_0\cdots M_p}\right], \end{aligned} \qquad (2.8)$$

where g_{ij} is the pull-back of the spacetime metric to the world-volume, i.e.

$$g_{ij} = \partial_i X^M \partial_j X^N G_{MN}\,, \qquad (2.9)$$

and $P[A^{(p+1)}]$ denotes the pull-back of the $(p+1)$-form potential, which is given explicitly in the second line. We use $\varepsilon_{i_0\cdots i_p}$ to denote the world-volume volume tensor. Hence,

$$\varepsilon_{012\cdots p} = \sqrt{-g} \qquad \text{and} \qquad \varepsilon^{012\cdots p} = -\frac{1}{\sqrt{-g}}\,. \qquad (2.10)$$

In passing, we should comment that the expression in eq. (2.8) is not the complete world-volume action. For example, we have dropped all of the fermions. It is, of course, consistent with the equations of motion to consider purely bosonic solutions as we will do in the following — the fermions will reappear in our discussion of supersymmetry in section 3. However, in the case of the M5-brane [8, 6] and the D3-brane [9, 10], there are additional bosonic world-volume fields: a self-dual three-form on the M5-brane, and a conventional gauge field on the D3-brane. Examining the full equations of motion confirms that for all of the brane configurations considered here, these additional fields are consistently set to zero. Hence we have verified that it is consistent to work with the reduced action (2.8) for the present analysis.

Following ref. [1], we now wish to find stable test brane solutions where an $(n-2)$-brane has expanded on the S^n to a sphere of fixed θ while it orbits the S^n in the ϕ direction. It is convenient to choose static gauge in the world-volume theory, where the world-volume coordinates σ^i are identified with the appropriate space-time coordinates:

$$\sigma_0 \equiv \tau = t\,,\ \sigma_1 = \chi_1\,,\ \ldots,\ \sigma_{n-2} = \chi_{n-2}\,. \tag{2.11}$$

We will consider a trial solution of the form

$$\theta = \text{constant}\,, \qquad \phi = \phi(\tau)\,, \qquad r = 0\,, \tag{2.12}$$

which corresponds to a spherical $(n-2)$-brane of radius $L\sin\theta$ moving around inside the S^n. From ref. [1] we know that ϕ will be linear in τ, i.e. $\dot{\phi}$ =constant, but we leave $\phi(\tau)$ arbitrary for the purpose of doing a hamiltonian analysis below.

For the particular embedding of the $(n-2)$-brane in eqs. (2.11) and (2.12), the pullback of the metric (2.9) is

$$g_{ij} = \begin{pmatrix} -1 + L^2\cos^2\theta\dot{\phi}^2 & 0 \\ 0 & L^2\sin^2\theta\,(g_\chi)_{ij} \end{pmatrix}, \tag{2.13}$$

where $(g_\chi)_{ij}$ denotes the metric on the unit $(n-2)$-sphere (2.5) (with, as in eq. (2.11), $\chi^i = \sigma^i$). Substituting the trial solution (2.12) into the world-volume action (2.8) and integrating over the angular coordinates, yields the following lagrangian[1]

$$\mathcal{L}_{n-2} = \frac{N}{L}\left[-\sin^{n-2}\theta\,\sqrt{1 - L^2\cos^2\theta\,\dot{\phi}^2} + L\sin^{n-1}\theta\,\dot{\phi}\right]. \tag{2.14}$$

Here we have introduced the (large positive) integer N using the quantization of the n-form flux on S^n. In each of the cases of interest, the flux is related to the tension of the corresponding brane by

$$A_{n-2}T_{n-2} = \frac{N}{L^{n-1}}\,, \tag{2.15}$$

[1]Note that we did not remain consistent with eq. (2.6) at this point for $n = 7$. That is, the sign of the second term in eq. (2.8) is reversed, which corresponds to the brane having the opposite charge. In our conventions, the brane expanding into S^7 is an *anti*-M5-brane.

where A_{n-2} is the area of a unit $(n-2)$-sphere. The momentum conjugate to ϕ becomes

$$P_\phi = N\left[\frac{L\sin^{n-2}\theta\cos^2\theta\,\dot\phi}{\sqrt{1-L^2\cos^2\theta\,\dot\phi^2}}+\sin^{n-1}\theta\right]. \tag{2.16}$$

We can invert this relation to write

$$\dot\phi = \frac{1}{L}\frac{p-\sin^{n-1}\theta}{\cos\theta\sqrt{p^2-2p\sin^{n-1}\theta+\sin^{2n-4}\theta}}, \tag{2.17}$$

where we have introduced $p = P_\phi/N$. The corresponding hamiltonian (or routhian) becomes

$$\mathcal{H}_n = P_\phi\dot\phi-\mathcal{L}_{n-2} = \frac{N}{L}\sqrt{p^2+\tan^2\theta\,(p-\sin^{n-3}\theta)^2}\,. \tag{2.18}$$

Given that the hamiltonian (2.18) (or the corresponding lagrangian (2.14)) is independent of ϕ, it is clear that equations of motion will be solved with the momentum (and hence $\dot\phi$) being constant. For fixed p, eq. (2.18) can be regarded as the potential that determines the equilibrium radius of the spherical membrane, i.e. fixing the angle θ. In general, one finds

$$\frac{\partial\mathcal{H}_n}{\partial\theta} \propto \sin\theta(\sin^{n-3}\theta-p)\left((n-3)\sin^{n-1}\theta-(n-2)\sin^{n-3}\theta+p\right). \tag{2.19}$$

As illustrated in figure 1, for $p\le 1$ and n even there are two degenerate minima, at $\sin\theta = 0$ and $\sin\theta = p^{1/n-3}$, separated by some intermediate maximum. At either of the minima, the energy evaluates to

$$\mathcal{H}_n = \frac{N}{L}p = \frac{P_\phi}{L}. \tag{2.20}$$

Of course, the minimum at $\sin\theta = p^{1/(n-3)}$ cannot exist for $p>1$ [1]. As p grows beyond $p=1$, the minimum at $\theta>0$ first lifts above the one at $\sin\theta = 0$, before disappearing completely for

$$p > 2\left(\frac{n-2}{n-1}\right)^{\frac{n-1}{2}}. \tag{2.21}$$

For the case of interest, $n=4$, this limit corresponds to $p\gtrsim 1.0887$.

For the case of n odd, the results are similar, as illustrated in figure 2. For $p\le 1$, there are now three degenerate minima, at $\sin\theta = 0$ and at $\sin\theta = \pm p^{1/(n-3)}$, separated by some intermediate maxima. Again at any of the minima, the energy is $\mathcal{H}_n = P_\phi/L$. As p grows beyond $p=1$, the minima at $\theta\ne 0$ are lifted above the one at $\sin\theta = 0$ and then disappear completely if p exceeds the bound given in eq. (2.21). For the cases of interest, $n=5$ and 7, this limit corresponds to $p>1.125$ and $p\gtrsim 1.1574$, respectively.

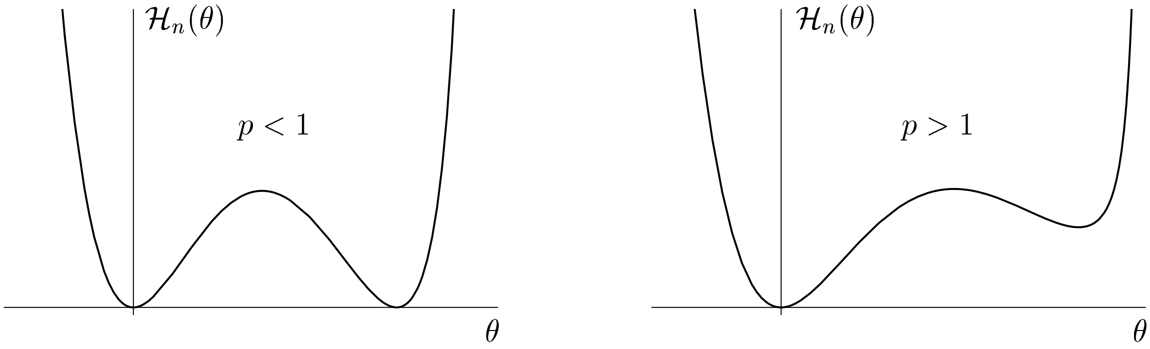

Figure 1: Energy of expanded $(n-2)$-brane, n even, as a function of its radius. For $p = P_\phi/N \leq 1$ (left figure) there are two degenerate minima. For $p > 1$ the second minimum acquires higher energy and eventually disappears completely.

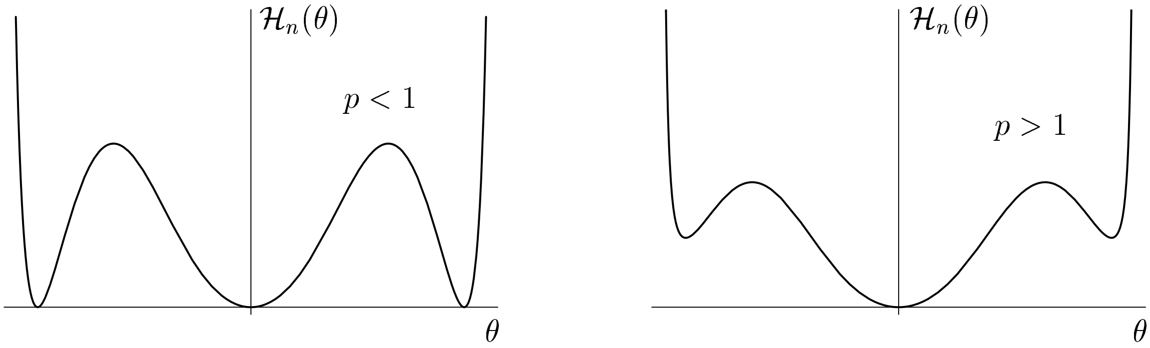

Figure 2: Energy of expanded $(n-2)$-brane, n odd, as a function of its radius. Same behavior as for n even, except that the potential is symmetrical.

We should comment on two related aspects of these results. First, in our figures we allow θ to take negative values. This range, $\theta < 0$, corresponds to the $(n-2)$-brane expanding into the n-sphere, but with the opposite orientation for the angular coordinates. For n even, this is equivalent to an anti-brane with momentum P_ϕ expanding into the n-sphere. For n odd, this is in fact the identical configuration of a brane expanding into the n-sphere. This explains a second curious point, namely, for n odd, the potential (2.18) is even under $\theta \to -\theta$, but there is no such symmetry for n even. In the first case, the configurations with $\theta < 0$ are actually redundant, being physically equivalent to those with $\theta > 0$. In contrast, for n even, positive and negative θ are distinct physical configurations, and our results show there are no stable expanded configurations for an anti-brane carrying (positive) angular momentum P_ϕ. The latter result also applies for the case of n odd, but requires an additional calculation. To describe an anti-brane, one would consider a test brane with the opposite charge. That is, one would reverse the sign of the second term in the action (2.8). Repeating the above analysis in this case results in a potential of the form

$$\overline{\mathcal{H}}_n = \frac{N}{L}\sqrt{p^2 + \tan^2\theta\left(p + \sin^{n-3}\theta\right)^2}\,. \tag{2.22}$$

As expected for n even, this potential is identical to that in eq. (2.18) up to $\theta \to -\theta$. On the other hand, for n odd, it is easy to see that the anti-brane potential (2.22) has no extrema except at $\theta = 0$.

From the point of view of m-dimensional supergravity in the AdS space, the stable brane configurations correspond to massive states with $M = P_\phi/L$. The motion on the S^n means that these states are also charged under a U(1) subgroup of the SO($n+1$) gauge symmetry in the reduced supergravity theory. With the appropriate normalizations, the charge is $Q = P_\phi/L$, and hence one finds that these configurations satisfy the appropriate BPS bound [1]. One can therefore anticipate that all of these configurations should be supersymmetric, and we confirm this result with an explicit construction of the residual supersymmetries in section 3.

It is also interesting to consider the motion of these stable configurations. Evaluating eq. (2.17) for any of the above solutions, remarkably one finds the same result: $\dot\phi = 1/L$, independent of P_ϕ! Note then that the center of mass motion for any of the configurations in the full $(m+n)$-dimensional background is along a null trajectory, since

$$ds^2 = -\left(1 - L^2\cos^2\theta\,\dot\phi^2\right)dt^2 = 0 \tag{2.23}$$

when evaluated for $\theta = 0$ and $\dot\phi = 1/L$. This is, of course, the expected result for a massless "point-like" graviton, but it applies equally well for the expanded brane configurations. However, note that in the expanded configurations, the motion of each element of the sphere is along a timelike trajectory, with $ds^2 = -\sin^2\theta\,dt^2$.

Naively, one might argue that the zero-size solution at $\sin\theta = 0$ is unphysical and that the true physical graviton should be identified with the expanded brane. In particular, one might observe that if one evaluates the expression (2.16) for the angular momentum by first taking $\dot\phi = 1/L$ and then $\theta = 0$, the result is $P_\phi = 0$. Examining more closely, however, it is easy to show that there is a limit $\dot\phi \to 1/L$ and $\theta \to 0$ such that P_ϕ remains finite. This is, of course, analogous to the case in ordinary relativistic mechanics, where a limiting procedure (with $m \to 0, v \to 1$) is required in order to see that zero-mass particles can carry finite momentum. In the present case, the limit is slightly unusual in that it involves approaching $\dot\phi = 1/L$ from above. That is, the center of mass velocity exceeds the speed of light. However, the motion of the elements of the sphere always remains subluminal! Hence such a discussion cannot rule out the point-like configuration as unphysical, and it seems like the zero-size state needs to be taken into account as well.

2.2 Giant gravitons in AdS

In the previous section, we have seen that a spherical $(n-2)$-brane configuration has the same quantum numbers as the point-like (super)graviton. Motivated by the analysis of ref. [11] (see discussion in section 4), one might also consider the possibility of a brane expanding into the AdS part of the spacetime. In this section, we will show that there is in fact a stable expanded $(m-2)$-brane configuration in the AdS space, which again carries the same quantum numbers as the point-like graviton.

In this case, we again begin with the same world-volume action (2.8). Now, however, we wish to find stable solutions where an $(m-2)$-brane has expanded into the AdS_m space to a sphere of constant r while it orbits in the ϕ direction on the S^n. Choosing static gauge, we identify

$$\sigma_0 \equiv \tau = t\,,\ \ \sigma_1 = \alpha_1\,,\ \ \ldots,\ \ \sigma_{m-2} = \alpha_{m-2}\,. \tag{2.24}$$

Our trial solution will be

$$\theta = 0\,, \qquad \phi = \phi(\tau)\,, \qquad r = \text{constant}\,. \tag{2.25}$$

Now one can calculate the pull-backs of the metric and the $(n-1)$-form potential, substitute the trial solution and integrate over the angular directions. The resulting lagrangian is

$$\mathcal{L}_{m-2} = A_{m-2}T_{m-2}\left[-r^{m-2}\sqrt{1+\frac{r^2}{\widetilde{L}^2}-L^2\dot{\phi}^2}+\frac{r^{m-1}}{\widetilde{L}}\right], \tag{2.26}$$

where A_{m-2} is the area of a unit S^{m-2}. Note that this action actually describes an *anti-brane* by the conventions in eqs. (2.7) and (2.8) — that is, the sign of the second term in the action (2.8) has been reversed, corresponding to choosing the opposite brane charge. As we will see, this choice is required to produce an expanded brane. As in eq. (2.14), it is convenient to introduce[2]

$$A_{m-2}T_{m-2} = \frac{\widetilde{N}}{L\,\widetilde{L}^{m-2}}\,. \tag{2.27}$$

The conjugate momentum for ϕ now becomes

$$P_\phi = \widetilde{N}\frac{r^{m-2}}{\widetilde{L}^{m-2}}\frac{L\,\dot{\phi}}{\sqrt{1+\frac{r^2}{\widetilde{L}^2}-L^2\dot{\phi}^2}}\,. \tag{2.28}$$

[2]Note that $\widetilde{N}$ is *not* the integer appearing in the background flux quantization condition (2.15) for the M-theory cases $m=4,7$. In terms of the $(m-2)$-brane tension, the latter becomes $A_{m-2}T_{m-2} = b(m)N^{\frac{m-3}{2}}/L\widetilde{L}^{m-2}$, where $b(4)=\sqrt{2}$ and $b(5)=1=b(7)$. This follows from the relation between the M2- and M5-brane tensions, $2\pi T_5 = T_2^2$, as well as eqs. (2.3) and (2.15).

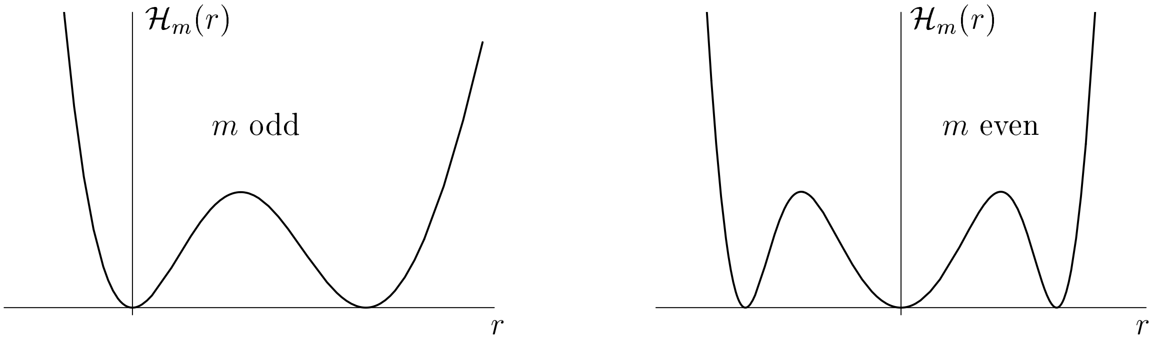

Figure 3: Energy of an $(m-2)$-brane expanding into AdS_m as a function of its radius.

Introducing the notation $\tilde{p} = P_\phi/\widetilde{N}$, one calculates the hamiltonian to be

$$\mathcal{H}_m = P_\phi\dot{\phi} - \mathcal{L}_{m-2} = \frac{\widetilde{N}}{L}\left[\sqrt{\left(1+\frac{r^2}{\widetilde{L}^2}\right)\left(\tilde{p}^2+\frac{r^{2m-4}}{\widetilde{L}^{2m-4}}\right)} - \frac{r^{m-1}}{\widetilde{L}^{m-1}}\right]. \tag{2.29}$$

Examining $\partial\mathcal{H}_m/\partial r = 0$, one finds minima located at

$$r = 0 \qquad \text{and} \qquad \left(\frac{r}{\widetilde{L}}\right)^{m-3} = \tilde{p}\,. \tag{2.30}$$

The energy at each of the minima is $\mathcal{H}_m = \widetilde{N}\tilde{p}/L = P_\phi/L$, matching the BPS mass found in the previous section. The potential for even and odd m is illustrated in figure 3. Note that for m even, there is a single minimum with $r > 0$, while for m odd, there are two such minima, one on either side of $r = 0$. The physical reasons for this structure are the same as in the case for the branes expanding on the n-sphere. Also as before, one can show that for the case of m odd, there are no static solutions (other than $r = 0$) for an anti-$(m-2)$-brane expanding into the AdS space. An essential difference from that case, however, is that the minima corresponding to expanded branes persist for arbitrarily large values of $\tilde{p}$.

As we did above, we consider the center of mass motion of these brane configurations. One again finds for any of the stable minima that $\dot{\phi} = 1/L$, independent of P_ϕ. The center of mass motion then follows a null trajectory in the full $(m+n)$-dimensional background spacetime for either the point-like state or the branes that have expanded into the AdS space. In the latter case, the motion of each element of the sphere is along a time-like trajectory with $ds^2 = -(r/\widetilde{L})^2\,dt^2$.

2.3 Instanton transitions

So far we have seen that for a given background $AdS_m \times S^n$, there are three potential brane configurations to describe a graviton carrying angular momentum P_ϕ: the "point-like" graviton, the giant graviton of ref. [1] consisting of a spherical

($n-2$)-brane expanded out into the n-sphere, and a "dual" giant graviton consisting of a spherical ($m-2$)-brane which expands out into the AdS space. Quantum mechanically one might expect these three states to mix, and so one is motivated to look for instanton solutions describing tunneling from one state to another. In this section we will derive explicit expressions for the instantons evolving between the expanded ($n-2$)-brane state and the zero-size state, as well as between the expanded ($m-2$)-brane state and the point-like state. It would be interesting to construct instantons describing tunneling directly between the two expanded brane configurations, but it is not clear that this can be done in the framework of test-branes (at least in the case of $D = 11$).

We start with the instanton for the ($n-2$)-brane in S^n. To begin, we extend the ansatz (2.12) to allow for a time dependent angle $\theta(\tau)$. The lagrangian (2.14) is then extended to

$$\mathcal{L}_{n-2} = \frac{N}{L}\left[-\sin^{n-2}\theta\sqrt{1 - L^2\cos^2\theta\,\dot{\phi}^2 - L^2\,\dot{\theta}^2} + L\sin^{n-1}\theta\,\dot{\phi}\right]. \tag{2.31}$$

Now we make a Legendre transform to eliminate $\dot{\phi}$ in favor of P_ϕ to produce a routhian $\mathcal{R}_n(P_\phi, \dot{\theta}, \theta)$. Next we analytically continue to euclidean time, $\tau \to iz$, which yields

$$\mathcal{R}_n^E = \frac{N}{L}\sqrt{1 + L^2(\theta')^2}\,V(\theta)\,, \tag{2.32}$$

$$V(\theta) \equiv \sqrt{p^2 + \tan^2\theta(p - \sin^{n-3}\theta)^2}\,, \tag{2.33}$$

where $\theta' = \partial_z\theta$. Rather than work with the second order euclidean equations of motion, it is easier to find the instanton solution by evaluating the corresponding conserved "energy",

$$\mathcal{H}^E = \frac{\delta\mathcal{R}_n^E}{\delta\theta'}\theta' - \mathcal{R}_n^E = -\frac{N}{L}\frac{V(\theta)}{\sqrt{1 + L^2(\theta')^2}}\,. \tag{2.34}$$

At the extrema, $\sin\theta = 0$ and $\sin^{n-3}\theta = p$, which will correspond to the instanton end-points, this evaluates to $\mathcal{H}^E = -Np/L$. Hence we simply need to solve

$$\frac{V(\theta)}{\sqrt{1 + L^2(\theta')^2}} = p\,, \tag{2.35}$$

which has the remarkably simple solutions

$$\sin^{m-3}\theta_\pm(z) = \frac{p}{1 + e^{\pm(m-3)(z-z_0)/L}}\,. \tag{2.36}$$

The two solutions correspond to the instanton (+) describing a transition from the expanded ($n-2$)-brane (at $z \to -\infty$) to the point-like configuration at ($z \to \infty$);

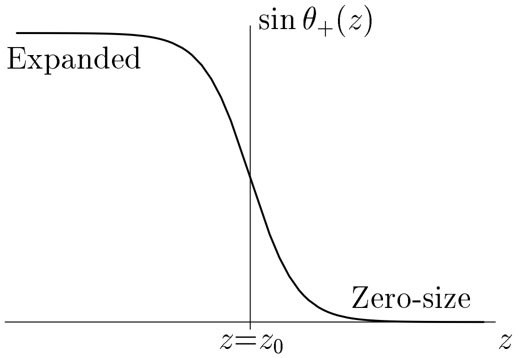

Figure 4: Instanton transition from the expanded $(n-2)$-brane on S^n to the collapsed one (plot made for $m = 4$).

and the anti-instanton $(-)$ describing the opposite transition. The constant z_0 is an integration constant giving the instanton position in euclidean time. See figure 4 for a plot of the solution.

Next one would like to calculate the euclidean action. Because $\mathcal{R}_n^E$ remains finite when evaluated at either of the extrema, evaluating $\int dz \mathcal{R}_n^E$ for the instanton configuration would yield an infinite result. Instead, the relevant quantity to evaluate is

$$\mathcal{S}_n^E = \int_{-\infty}^{\infty} dz \, \left(\mathcal{R}_n^E\left(\theta_{\text{instanton}}\right) - \mathcal{R}_n^E\left(\theta_{\text{end-point}}\right) \right) , \tag{2.37}$$

which will give a measure of the degree of tunneling. Using eq. (2.35), we find

$$\Delta\mathcal{R}_n^E \equiv \mathcal{R}_n^E(\theta) - \mathcal{R}_n^E(\theta = 0) = \frac{N}{Lp}\left[V(\theta)^2 - p^2\right] , \tag{2.38}$$

and the total action evaluates to

$$\begin{aligned} \mathcal{S}_n^E &= N \int_0^{\arcsin(p^{1/(n-3)})} d\theta \, \sqrt{V(\theta)^2 - p^2} \\ &= N \int_0^{\arcsin(p^{1/(n-3)})} d\theta \, \tan\theta \left(p - \sin^{n-3}\theta\right) . \end{aligned} \tag{2.39}$$

We can evaluate this explicitly for the cases of interest:

$$\mathcal{S}_{n=4}^E = \frac{N}{2}\left[(1-p)\ln(1-p) - (1+p)\ln(1+p) + 2p\right] , \tag{2.40}$$

$$\mathcal{S}_{n=5}^E = \frac{N}{2}\left[(1-p)\ln(1-p) + p\right] , \tag{2.41}$$

$$\mathcal{S}_{n=7}^E = \frac{N}{2}\left[(1-p)\ln(1-\sqrt{p}) + \sqrt{p} + p/2\right] . \tag{2.42}$$

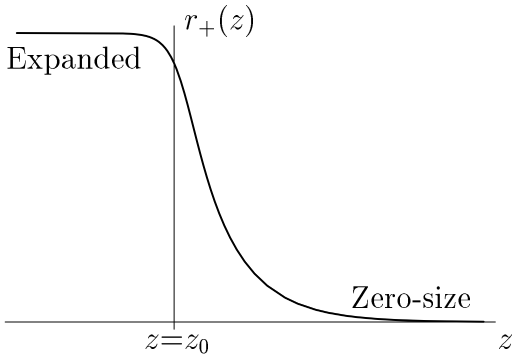

Figure 5: Instanton transition from the expanded $(m-2)$-brane on AdS_m to the collapsed one (plot made for $m = 7,\ \tilde{p} = 1$).

In all the cases these are monotonically increasing functions of p, with finite limits as $p \to 1$.

We can repeat the analysis to find instantons for the $(m-2)$-brane which expands on the AdS space. One begins in this case by allowing a time dependent radius, $r(\tau)$. In terms of the scaled radius, $\tilde{r} = r/\widetilde{L}$, the euclidean routhian is

$$\mathcal{R}^E_m = \frac{\widetilde{N}}{L}\left[\sqrt{1+\frac{\widetilde{L}^2(\tilde{r}')^2}{(1+\tilde{r}^2)^2}}\sqrt{(1+\tilde{r}^2)(\tilde{p}^2+\tilde{r}^{2m-4})} - \tilde{r}^{m-1}\right]. \tag{2.43}$$

Proceeding as above, we derive a first order equation for the instanton, which may be written as

$$\widetilde{L}\tilde{r}' = \pm\frac{\tilde{r}(1+\tilde{r}^2)(\tilde{p}-\tilde{r}^{m-3})}{\tilde{p}+\tilde{r}^{m-1}}\,. \tag{2.44}$$

In the cases of interest, it is relatively straightforward to integrate this equation to yield the solution

$$e^{\pm 2(z-z_0)/L} = \left(1+\tilde{r}^{-2}\right)^{\frac{m-3}{2}}\left(\tilde{p}-\tilde{r}^{m-3}\right). \tag{2.45}$$

These provide implicit solutions (which can be solved explicitly) for the desired instantons, evolving between the expanded $(m-2)$-brane and the point-like one, see figure 5.

In evaluating the euclidean action, we use eq. (2.44) to find

$$\Delta\mathcal{R}^E_m \equiv \mathcal{R}^E_m(\theta) - \mathcal{R}^E_m(\theta=0) = \frac{\widetilde{N}}{L}\frac{\tilde{r}^2(\tilde{p}-\tilde{r}^{m-3})^2}{\tilde{p}+\tilde{r}^{m-1}}\,, \tag{2.46}$$

and the total action becomes

$$\mathcal{S}^E_m = \frac{\widetilde{N}\widetilde{L}}{L}\int_0^{p^{1/(m-3)}} d\tilde{r}\,\tilde{r}\,\frac{\tilde{p}-\tilde{r}^{m-3}}{1+\tilde{r}^2} = \frac{\widetilde{N}\widetilde{L}}{2L}\int_0^{p^{2/(m-3)}} du\,\frac{p-u^{(m-3)/2}}{1+u}\,. \tag{2.47}$$

Explicitly for AdS_4, AdS_5, and AdS_7, we find

$$\mathcal{S}^E_{m=4} = \frac{\sqrt{2N}}{4}\left[\tilde{p}\ln(1+\tilde{p}^2) + 2\arctan\tilde{p} - 2\tilde{p}\right], \tag{2.48}$$

$$\mathcal{S}^E_{m=5} = \frac{N}{2}\left[(1+\tilde{p})\ln(1+\tilde{p}) - \tilde{p}\right], \tag{2.49}$$

$$\mathcal{S}^E_{m=7} = N^2\left[(\tilde{p}-1)\ln(1+\sqrt{\tilde{p}}) + \sqrt{\tilde{p}} - \tilde{p}/2\right]. \tag{2.50}$$

It is no coincidence that these expressions resemble those following from eq. (2.39) — introducing $u = \sin^2\theta$ in the latter produces an integrand identical to the one in eq. (2.47), except that the denominator is replaced by $1-u$.

3. SUSY versus Goliath

3.1 Supersymmetry for giants

In this subsection we make a detailed analysis of the supersymmetries preserved by the expanded M2-brane configuration on $AdS_7 \times S^4$ described above, and we confirm the expectation that (for $p \le 1$) it preserves the same supersymmetries as the corresponding "point-like" supergraviton. Our conventions here largely follow those of the review by Duff [6].

Eleven-dimensional supergravity is described by the vielbein $E_M{}^A(X)$, the gravitino $\Psi_M(X)$, and the three-form potential $A^{(3)}_{MNP}(X)$ with field strength $F^{(4)}_{MNPQ}$. Also, in a superspace formulation, one introduces a six-form $A^{(6)}_{MNPQRS}$ whose field strength is dual to $F^{(4)}_{MNPQ}$. For the $AdS_7 \times S^4$ compactification, we are considering the Freund-Rubin ansatz [7] with the four-form $F^{(4)} = 3/L\,\varepsilon(S^4)$, where $\varepsilon(S^4)$ is the volume form on the four-sphere. The spinors are Majorana, and the gamma-matrices satisfy $\{\Gamma^A,\Gamma^B\} = 2\eta_{AB}$ with $\eta^{AB} = \mathrm{diag}(-,+,\ldots,++)$. We also adopt the standard notation: $\Gamma^{AB\cdots C} = \Gamma^{[A}\Gamma^B\cdots\Gamma^{C]}$. (We only write eleven-dimensional gamma-matrices in the following.) The SUSY transformation of the gravitino is

$$\delta\Psi_M = \tilde{D}_M\epsilon - \frac{1}{288}\left(\Gamma_M{}^{PQRS} - 8\delta_M{}^P\Gamma^{QRS}\right)F_{PQRS}\epsilon\,, \tag{3.1}$$

where $\tilde{D}_M$ is a supercovariant derivative containing the usual connection augmented by gravitino-dependent terms. A bosonic configuration ($\Psi_M = 0$) will respect the supersymmetry for parameters ϵ such that $\delta\Psi_M = 0$ (since the bosonic fields vary into the gravitino).

For the $AdS_7 \times S^4$ background, $\Psi_M = 0$ and so the supercovariant derivative reduces to an ordinary covariant derivative in eq. (3.1). Thus in this background, the Killing spinor equations ($\delta\Psi_M = 0$) become

$$\left(D_m - \frac{1}{2L}\gamma\Gamma_m\right)\epsilon = 0\,; \qquad \left(D_\mu - \frac{1}{4L}\gamma\Gamma_\mu\right)\epsilon = 0\,; \tag{3.2}$$

for m and μ on S^4 and AdS_7, respectively. Implicitly here and in the following, we are using the natural orthonormal frame pointing along the coordinate directions for the metrics given in eqs. (2.1) and (2.2). Recall that the AdS_7 coordinates are $t, r, \alpha_1, \ldots, \alpha_5$, and those on S^4 are $\theta, \phi, \chi_1, \chi_2$. In eq. (3.2), $\gamma \equiv \Gamma^{\theta\phi\chi_1\chi_2}$. The Killing spinors may then be written as (see also ref. [12])

$$\epsilon(X) = e^{\frac{1}{2}\theta\gamma\Gamma^\theta} e^{\frac{1}{2}\phi\gamma\Gamma^\phi} e^{-\frac{1}{2}\chi_1\Gamma^{\chi_1\theta}} e^{-\frac{1}{2}\chi_2\Gamma^{\chi_2\chi_1}} \eta\,, \tag{3.3}$$

where

$$\eta(t, r, \alpha_1, \ldots, \alpha_5) = e^{\frac{1}{2}\alpha\Gamma^r\gamma} e^{-\frac{t}{4L}\Gamma^t\gamma} e^{\frac{1}{2}\alpha_1\Gamma^{\alpha_1 r}} e^{\frac{1}{2}\alpha_2\Gamma^{\alpha_2\alpha_1}} \cdots e^{\frac{1}{2}\alpha_5\Gamma^{\alpha_5\alpha_4}} \epsilon_0\,. \tag{3.4}$$

Here $\alpha = \sinh^{-1}(r/\tilde{L}) = \sinh^{-1}(r/2L)$. Note that the "AdS gamma matrix" factors in eq. (3.4) commute with the "four-sphere gamma matrix" factors in eq. (3.3), and so one is free to re-order these exponentials. Finally in eq. (3.4), ϵ_0 is an arbitrary constant spinor, and thus one has that the $AdS_7 \times S^4$ background is maximally supersymmetric, preserving 32 supersymmetries.

Now in curved superspace with coordinates $Z^M = (X^m, \Theta^\mu)$ and eleven-dimensional supergravity described by the super-vielbein $E_M{}^A(Z)$ and super-three-form $A^{(3)}_{ABC}(Z)$, the M2-brane action is given by

$$S = T_2 \int d^3\sigma \left[-\frac{1}{2}\sqrt{-g} g^{ij} E_i{}^a E_j{}^b \eta_{ab} + \frac{1}{2}\sqrt{-g} + \frac{1}{3!}\varepsilon^{ijk} E_i{}^A E_j{}^B E_k{}^C A^{(3)}_{CBA} \right], \tag{3.5}$$

where the pull-back is defined by $E_i{}^A = \partial_i Z^M E_M{}^A$ and $A = (a, \alpha)$ are tangent space indices. The action is invariant under target-space supersymmetry and, when the supergravity constraints are satisfied, under κ-symmetry. The bosonic part of the action reduces to that given in eq. (2.8) if we substitute in the solution for the world-volume metric equations of motion,

$$g_{ij} = \partial_i X^M \partial_j X^M G_{MN}(X)\,. \tag{3.6}$$

For the brane configuration studied in the section 2.1, we have set the spinor coordinates $\Theta = 0$, as well as having set the gravitino field $\Psi_M = 0$ in the background spacetime. This configuration will have residual supersymmetry if the following combined target-space supersymmetry and κ-symmetry transformations can be satisfied for some spinor(s) $\epsilon(X)$:

$$\delta\Psi_M = \left[D_M - \frac{1}{288}\left(\Gamma_M{}^{PQRS} - 8\delta_M{}^P \Gamma^{QRS}\right) F_{PQRS} \right] \epsilon(X) = 0\,, \tag{3.7}$$

$$\delta\Theta = \epsilon(X)|_{M2} + (1+\Gamma)\kappa(\sigma) = 0\,, \tag{3.8}$$

where

$$\Gamma = -\frac{1}{3!}\varepsilon^{ijk}\partial_i X^M \partial_j X^N \partial_k X^P \Gamma_{MNP}\,. \tag{3.9}$$

The first of these constraints (3.7) are, of course, the background Killing spinor equations, which have the solutions given in eqs. (3.3) and (3.4). Note that the second equation is only evaluated (defined) on the M2-brane world-volume.

The usual strategy [13], which we follow here, is to first construct the Killing spinors (given above) and then enforce the second equation. After imposing the κ-symmetry gauge choice $(1+\Gamma)\Theta = 0$ and requiring that it be preserved by the transformation (3.8), one finds a relation between ϵ and κ. Using this, the second constraint (3.8) is rewritten as

$$\Gamma\epsilon = \epsilon\,, \tag{3.10}$$

which again is only evaluated at the position of the M2-brane.

For the particular embedding of the membrane described in eqs. (2.11) and (2.12) we find

$$\begin{aligned}\Gamma &= \frac{1}{\sqrt{-g}}\,\partial_\tau X^M \partial_{\sigma_1} X^N \partial_{\sigma_2} X^P\, E^A{}_M E^B{}_N E^C{}_P\, \Gamma_{ABC} \\ &= \frac{1}{L^2\sin^3\theta\sin\chi_1}\left[E^{\hat t}{}_t E^{\hat\chi_1}{}_{\chi_1} E^{\hat\chi_2}{}_{\chi_2}\,\Gamma_{\hat t\hat\chi_1\hat\chi_2} + \dot\phi E^{\hat\phi}{}_\phi E^{\hat\chi_1}{}_{\chi_1} E^{\hat\chi_2}{}_{\chi_2}\,\Gamma_{\hat\phi\hat\chi_1\hat\chi_2}\right] \\ &= -\frac{1}{\sin\theta}\left[\Gamma^{t\chi_1\chi_2} - \cos\theta\,\Gamma^{\phi\chi_1\chi_2}\right].\end{aligned} \tag{3.11}$$

We have employed (and then dropped) hatted indices to distinguish between local and tangent space indices. Also as stated above, we are using the natural vielbein directed along the coordinate directions, and so $E^A{}_M$ can be read off from the metrics in eqs. (2.1) and (2.2). It is straightforward to check that $\Gamma^2 = 1$. Note that in going between the second and third line in eq. (3.11), we have substituted $\dot\phi = 1/L$, which is satisfied for both of the stable brane configurations.

Now we write the condition in (3.10) as

$$\left[\Gamma^{t\chi_1\chi_2} - \cos\theta\Gamma^{\phi\chi_1\chi_2} + \sin\theta\right]\epsilon = 0\,, \tag{3.12}$$

or

$$-\Gamma^\theta\gamma\left[\Gamma^{t\phi} + \cos\theta - \sin\theta\gamma\Gamma^\theta\right]\epsilon = -\Gamma^\theta\gamma\left[\Gamma^{t\phi} + e^{-\theta\gamma\Gamma^\theta}\right]\epsilon = 0\,. \tag{3.13}$$

Substituting in the Killing spinors (3.3) and (3.4), we must satisfy

$$\begin{aligned}0 &= \left[\Gamma^{t\phi} + e^{-\theta\gamma\Gamma^\theta}\right] e^{\frac12\theta\gamma\Gamma^\theta} e^{\frac12\phi\gamma\Gamma^\phi} e^{-\frac12\chi_1\Gamma^{\chi_1\theta}} e^{-\frac12\chi_2\Gamma^{\chi_2\chi_1}} e^{\frac12\alpha\Gamma^r\gamma} e^{-\frac{t}{4L}\Gamma^t\gamma}\cdots\epsilon_0 \\ &= e^{-\frac12\theta\gamma\Gamma^\theta}\left[\Gamma^{t\phi}+1\right] e^{\frac12\phi\gamma\Gamma^\phi} e^{-\frac12\chi_1\Gamma^{\chi_1\theta}} e^{-\frac12\chi_2\Gamma^{\chi_2\chi_1}} e^{\frac12\alpha\Gamma^r\gamma} e^{-\frac{t}{4L}\Gamma^t\gamma}\cdots\epsilon_0\,.\end{aligned} \tag{3.14}$$

In the second line above, we have used the fact that $\Gamma^{t\phi}$ and $\gamma\Gamma^\theta$ anticommute, and so

$$\Gamma^{t\phi} e^{\frac12\theta\gamma\Gamma^\theta} = e^{-\frac12\theta\gamma\Gamma^\theta}\,\Gamma^{t\phi}\,. \tag{3.15}$$

Now the remaining exponentials obviously commute with $\Gamma^{t\phi}$, except for the one containing $\Gamma^r\gamma$. However, the test brane configurations sit at $r = \tilde L\sinh\alpha = 0$, and so this factor reduces to the identity when evaluated on the M2-brane world-volume. Thus, the final condition (3.14) amounts to imposing

$$(\Gamma^{t\phi}+1)\epsilon_0 = 0\,. \tag{3.16}$$

That is, supersymmetry will be preserved with the constant spinors ϵ_0 satisfying this condition. We note that $(\Gamma^{t\phi})^2 = 1$ and $\mathrm{tr}(\Gamma^{t\phi}) = 0$ so that the spherical M2-brane preserves half of the supersymmetries of eleven-dimensional supergravity.

Note that the equation $\dot\phi = 1/L$, which was used in eq. (3.11), was crucial to this derivation of the final constraint (3.16). Thus the supersymmetry is preserved by precisely the two M2-brane solutions sitting at the stable minima of the potential $\mathcal{H}_{n=4}$ given in eq. (2.18). So both the expanded brane and the point-like state are BPS configurations, and they both preserve precisely the same supersymmetries. These configurations are supposed to describe a massless particle (e.g. a graviton) moving along the ϕ direction in the eleven-dimensional spacetime. The projection (3.16) is in accord with what one might have expected by examining the supersymmetry of gravitational waves propagating in flat space — see, for example, ref. [14].

3.2 Supersymmetry for "dual" giants

We consider now the situation of an M5-brane moving on $AdS_7\times S^4$, where the brane expands into the AdS_7 part of the space. We shall show that the expanded M5-brane preserves precisely the same supersymmetries as the M2-brane configurations.

The residual supersymmetries for a purely bosonic M5-brane configuration must again satisfy two constraints, one being that for the spacetime Killing spinors (3.7) and the other involving combined supersymmetry and κ-symmetry transformations on the world-volume. The full analysis is slightly more complicated for the M5-brane because of the self-dual three form in the world-volume theory — see, for example, ref. [8, 6]. However, when the self-dual three-form on the world-volume vanishes, as it does for the configurations of interest, the final result is that we must find Killing spinors that now satisfy the constraint

$$\Gamma\epsilon = \epsilon\,, \tag{3.17}$$

where

$$\Gamma = -\frac{1}{6!}\varepsilon^{i_1\cdots i_6}\partial_{i_1}X^{M_1}\cdots\partial_{i_6}X^{M_6}\Gamma_{M_1\cdots M_6}\,. \tag{3.18}$$

Given the embedding in eqs. (2.24) and (2.25) with $m=7$, we obtain from the corresponding induced metric

$$\sqrt{-g} = \sqrt{1+\frac{r^2}{4L^2}-L^2\dot\phi^2}\; r^5\sin^4\alpha_1\sin^3\alpha_2\cdots\sin\alpha_4\,, \tag{3.19}$$

using $\tilde{L} = 2L$ for the present background. Correspondingly, we find

$$\Gamma = \frac{1}{\sqrt{1+\frac{r^2}{4L^2} - L^2\dot{\phi}^2}} \left[-\sqrt{1+\frac{r^2}{4L^2}}\Gamma^{t\alpha_1\cdots\alpha_5} + L\dot{\phi}\,\Gamma^{\phi\alpha_1\cdots\alpha_5} \right]. \tag{3.20}$$

With $\dot{\phi} = 1/L$, the condition (3.17) can be written as

$$\left[\frac{r}{2L} + \sqrt{1+\frac{r^2}{4L^2}}\Gamma^{t\alpha_1\cdots\alpha_5} - \Gamma^{\phi\alpha_1\cdots\alpha_5} \right] \epsilon = 0\,. \tag{3.21}$$

At this point, we note that the product of all of the eleven-dimensional gamma matrices yields the identity, i.e. $\Gamma^{tr\alpha_1\cdots\alpha_5\theta\phi\chi_1\chi_2} = 1$. This allows us to write: $\Gamma^{t\alpha_1\cdots\alpha_5} = -\Gamma^r\gamma$ and $\Gamma^{\phi\alpha_1\cdots\alpha_5} = \Gamma^r\gamma\,\Gamma^{t\phi}$ where as above $\gamma = \Gamma^{\theta\phi\chi_1\chi_2}$. It is also useful to introduce $\sinh\alpha = \frac{r}{2L}$ as in eq. (3.4). Then the constraint (3.21) reduces to

$$\left[e^{-\alpha\Gamma^r\gamma} + \Gamma^{t\phi} \right] \epsilon = 0\,. \tag{3.22}$$

Now given the Killing spinors in eqs. (3.3) and (3.4), one can verify that this condition amounts to

$$\left(\Gamma^{t\phi} + 1 \right) \epsilon_0 = 0 \tag{3.23}$$

precisely as in the previous section. In deriving this final result, one uses that $\Gamma^{t\phi}$ and $\Gamma^r\gamma$ anticommute and that the brane configurations of interest sit at $\theta = 0$.

3.3 Gravitons on $AdS_4 \times S^7$

In the following two subsections we repeat the above analysis for two other cases of interest, M5-branes expanding on S^7 and M2-branes expanding on AdS_4. For the present compactification the Freund-Rubin ansatz [7] sets $F^{(4)} = \frac{6}{L}\varepsilon(AdS_4)$ where $\varepsilon(AdS_4)$ is the volume form on AdS_4. The Killing spinor conditions now become

$$\left(D_m - \frac{1}{2L}\gamma\Gamma_m \right) \epsilon = 0\,, \qquad \left(D_\mu + \frac{1}{2\tilde{L}}\Gamma_\mu\gamma \right) \epsilon = 0 \tag{3.24}$$

on S^7 and AdS_4, respectively, with the new definition $\gamma \equiv \Gamma^{tr\alpha_1\alpha_2}$. The solutions have a similar form to those found previously in eqs. (3.3) and (3.4):

$$\begin{aligned} \epsilon = e^{\frac{1}{2}\theta\gamma\Gamma^\theta} e^{\frac{1}{2}\phi\gamma\Gamma^\phi} e^{-\frac{1}{2}\chi_1\Gamma^{\chi_1\theta}} \prod_{j=2}^{5} e^{-\frac{1}{2}\chi_j\Gamma^{\chi_j\chi_{j-1}}} \times \\ \times e^{-\frac{\alpha}{2}\Gamma^r\gamma} e^{\frac{t}{L}\Gamma^t\gamma} e^{-\frac{\alpha_1}{2}\Gamma^{\alpha_1 r}} e^{-\frac{\alpha_2}{2}\Gamma^{\alpha_2\alpha_1}} \epsilon_0\,, \end{aligned} \tag{3.25}$$

where α is now defined by $\sinh\alpha = 2r/L$. Again ϵ_0 is an arbitrary 32-component constant spinor, and so $AdS_4 \times S^7$ provides another maximally supersymmetric background for the eleven-dimensional supergravity.

3.3.1 Giant gravitons in S^7

The analysis of the supersymmetry is similar to the case of the M2-branes on S^4. The M5-branes are embedded as in eqs. (2.11) and (2.12) with $n = 7$. The induced metric yields $\sqrt{-g} = L^5 \sin^6\theta \sin^4\chi_1 \cdots \sin\chi_4$ when evaluated for $\dot{\phi} = 1/L$. Then using appropriate expressions for the vielbein, eq. (3.18) yields

$$\Gamma = -\frac{1}{\sin\theta}\left(\Gamma^{t\chi_1\cdots\chi_5} - \cos\theta\Gamma^{\phi\chi_1\cdots\chi_5}\right). \tag{3.26}$$

In this case, we impose the supersymmetry condition $\Gamma\epsilon = \epsilon$ which can be manipulated to the form

$$\left(\Gamma^{t\phi} + e^{-\theta\gamma\Gamma^\theta}\right)\epsilon = 0\,. \tag{3.27}$$

It is satisfied by the Killing spinors (3.25) provided

$$\left(\Gamma^{t\phi} + 1\right)\epsilon_0 = 0\,. \tag{3.28}$$

3.3.2 Giant gravitons in AdS_4

The counterpart of the previous case is that of the M2-brane expanding in AdS_4. The embedding is now as in eqs. (2.24) and (2.25), and the induced metric leads to $\sqrt{-g} = R^3/\tilde{L}\sin^2\alpha_1\sin\alpha_2$. The supersymmetry condition is now $\Gamma\epsilon = -\epsilon$ as required for anti-M2-branes — compare to eq. (3.10) in section 3.1. The constraint becomes

$$\left[\sqrt{1+\frac{4r^2}{L^2}}\Gamma^{t\alpha_1\alpha_2} - \Gamma^{\phi\alpha_1\alpha_2} - \frac{2r}{L}\right]\epsilon = 0\,, \tag{3.29}$$

substituting $\tilde{L} = L/2$. Using $\sinh\alpha = 2r/L$, we have

$$\Gamma^{t\alpha_1\alpha_2}\left[e^{-\alpha\Gamma^{t\alpha_1\alpha_2}} + \Gamma^{t\phi}\right]\epsilon = 0\,. \tag{3.30}$$

Now using $\Gamma^{t\alpha_1\alpha_2} = -\Gamma^r\gamma$, the condition becomes

$$\left[e^{\alpha\Gamma^r\gamma} + \Gamma^{t\phi}\right]\epsilon = 0\,, \tag{3.31}$$

and using the explicit form of the Killing spinors in eq. (3.25), it reduces once more to the same condition found above,

$$\left(\Gamma^{t\phi} + 1\right)\epsilon_0 = 0\,. \tag{3.32}$$

Some comments on the supersymmetry projections used here are in order. Note that for the spherical M2-branes on S^4, the supersymmetry condition was $\Gamma\epsilon = \epsilon$ in section 3.1, while for the M2-branes expanding into AdS_4, we imposed the condition $\Gamma\epsilon = -\epsilon$ above. These choices are in agreement with the calculations in section 2. Recall that, as remarked after eq. (2.26), with our conventions while we have M2-branes expanding on S^4, the "dual" giant gravitons expanding in AdS_4 are anti-M2-branes. With regard to the residual supersymmetries, the difference between branes and anti-branes amounts to reversing a projection in the relevant κ-symmetry transformations. This in turn results in reversing the sign of the final constraint imposed on the Killing spinors. Hence we find the opposite supersymmetry constraints are satisfied for the expanded M2-branes on S^4 here and those on AdS_4 in section 3.1. In contrast, the projection imposed for the expanding M5-branes took the form $\Gamma\epsilon = \epsilon$ for both AdS_7 and S^7. This is also in accord with the results of section 2, where we found that both cases corresponded to anti-M5-branes within our conventions — see the footnote after eq. (2.14) — and hence the same supersymmetry constraint applies in both cases.

3.4 Gravitons on $AdS_5 \times S^5$

Finally, we consider D3-branes propagating in a ten-dimensional type-IIb background compactified on $AdS_5 \times S^5$. Ten-dimensional, type-IIb supergravity is described by the vielbein, a complex Weyl gravitino, a real four-form $A^{(4)}_{MNPQ}$ with *self-dual* field strength $F^{(5)}_{MNPQR}$, a complex two-form $A^{(2)}_{MN}$, a complex spinor Λ and a complex scalar Φ. In the $AdS_5 \times S^5$ background, we have $\Phi = A^{(2)}_{MN} = \Lambda = \Psi_M = 0$. The five-form field strength is $F = \frac{4}{L}[\varepsilon(AdS_5) + \varepsilon(S^5)]$. Given that the scalar and two-form vanish in this background, the supersymmetry variation of the complex spinor automatically vanishes, $\delta\Lambda = 0$. Hence to examine the background supersymmetries, the only non-trivial variation which needs to be considered is that of the gravitino. For the given background, the variation of the gravitino takes the form

$$\delta\Psi_M = D_M\epsilon - \frac{i}{480}\Gamma_M{}^{PQRST}F_{PQRST}\epsilon\,. \tag{3.33}$$

Demanding $\delta\Psi_M = 0$ leads to the Killing spinor equations

$$D_M\epsilon - \frac{i}{4}\left(\Gamma^{tr\alpha_1\alpha_2\alpha_3} + \Gamma^{\theta\phi\chi_1\chi_2\chi_3}\right)\Gamma_M\epsilon = 0\,. \tag{3.34}$$

We recall that here ϵ is a complex Weyl spinor satisfying $\Gamma^{11}\epsilon = \epsilon$ where $\Gamma^{11} = \Gamma^{tr\alpha_1\alpha_2\alpha_3\theta\phi\chi_1\chi_2\chi_3}$. (Our gamma matrices are now ten dimensional.) The Killing spinor condition can be rewritten then as

$$\begin{aligned} D_\mu\epsilon - \frac{i}{2}\gamma_{\mathrm{AdS}}\Gamma_\mu\epsilon = 0\,, &\qquad \gamma_{\mathrm{AdS}} = \Gamma^{tr\alpha_1\alpha_2\alpha_3} \equiv \gamma\,, \\ D_m\epsilon - \frac{i}{2}\gamma_5\Gamma_m\epsilon = 0\,, &\qquad \gamma_5 = \Gamma^{\theta\phi\chi_1\chi_2\chi_3}\,, \end{aligned} \tag{3.35}$$

on AdS_5 and S^5, respectively. The Killing spinor solutions are now

$$\begin{aligned} \epsilon = {} & e^{\frac{i}{2}\theta\gamma_5\Gamma^\theta} e^{\frac{i}{2}\phi\gamma_5\Gamma^\phi} e^{-\frac{i}{2}\chi_1\Gamma^{\chi_1\theta}} e^{-\frac{i}{2}\chi_2\Gamma^{\chi_2\chi_1}} e^{-\frac{i}{2}\chi_3\Gamma^{\chi_3\chi_2}} \times \\ & \times e^{i\frac{\alpha}{2}\Gamma^r\gamma} e^{-i\frac{t}{2L}\Gamma^t\gamma} e^{\frac{\alpha_1}{2}\Gamma^{\alpha_1 r}} e^{\frac{\alpha_2}{2}\Gamma^{\alpha_2\alpha_1}} e^{\frac{\alpha_3}{2}\Gamma^{\alpha_3\alpha_2}}\epsilon_0\,, \end{aligned} \tag{3.36}$$

where $\sinh\alpha = r/L$. Hence we have a maximally supersymmetric solution of the type-IIb supergravity equations with 32 residual supersymmetries.

3.4.1 Giant gravitons in S^5

Supersymmetric world-volume actions have been constructed for all type-II Dp-branes in a general supergravity background [10], but we will not elaborate on any of the details here. The analysis for determining the residual supersymmetries of a D-brane configuration is similar to that for their M-theory cousins. For our D3-brane configurations (in which the world-volume gauge fields vanish), residual supersymmetries are again determined by imposing a constraint on the background Killing spinors of the form $\Gamma\epsilon = \pm\epsilon$, now using the matrix

$$\Gamma = -\frac{i}{5!}\varepsilon^{i_1\cdots i_5}\partial_{i_1}X^{M_1}\cdots\partial_{i_5}X^{M_5}\Gamma_{M_1\cdots M_5}\,. \tag{3.37}$$

For the giant gravitons on S^5, we consider the embedding of a D3-brane as in eqs. (2.11) and (2.12). With this configuration, one finds

$$\Gamma = -\frac{i}{\sin\theta}\left(\Gamma^{t\chi_1\chi_2\chi_3} - \cos\theta\Gamma^{\phi\chi_1\chi_2\chi_3}\right), \tag{3.38}$$

and the condition $\Gamma\epsilon = \epsilon$ becomes, after pulling out a factor of $\Gamma^{\phi\chi_1\chi_2\chi_3}$,

$$\left(e^{-i\theta\gamma_5\Gamma^\theta} + \Gamma^{t\phi}\right)\epsilon = 0\,. \tag{3.39}$$

It is again easy to verify that the various exponentials in the Killing spinors (3.36) can be pulled through so as to reduce the condition to

$$\left(\Gamma^{t\phi} + 1\right)\epsilon_0 = 0\,. \tag{3.40}$$

3.4.2 Giant gravitons in AdS_5

We consider now the embedding as in eqs. (2.24) and (2.25). In the present case, the matrix Γ takes the form

$$\Gamma = \frac{i}{\sinh\alpha}\left[\cosh\alpha\Gamma^{t\alpha_1\alpha_2\alpha_3} - \Gamma^{\phi\alpha_1\alpha_2\alpha_3}\right]. \tag{3.41}$$

Now we impose the condition for preservation of supersymmetry, $\Gamma\epsilon = -\epsilon$, which corresponds to anti-D3-branes in our conventions.[3] This equation can be written as

$$\left[e^{-i\alpha\Gamma^r\gamma} + \Gamma^{t\phi}\right]\epsilon = 0 \tag{3.42}$$

with the same conclusion concerning the preservation of supersymmetry, namely, we project the constant spinors with

$$\left(\Gamma^{t\phi} + 1\right)\epsilon_0 = 0\,. \tag{3.43}$$

[3] By the conventions of section 2, this is again a case where branes expand in S^5 but anti-branes expand in AdS_5.

4. Discussion

For a given background $AdS_m \times S^n$, we have identified three different test-brane configurations carrying angular momentum P_ϕ, all of which have the same energy and preserve precisely the same supersymmetries. These configurations are the giant graviton of ref. [1] consisting of a spherical $(n-2)$-brane expanding out into the n-sphere, a "dual" giant graviton consisting of a spherical $(m-2)$-brane which expands out into the AdS space, and the point-like brane at the origin. The new configuration uncovered here is the "dual" giant graviton which expands into the AdS space. Our motivation for looking for these configurations came from the analysis of ref. [11]. There it was shown that in type-IIa superstring theory, a collection of D0-branes in an *electric* four-form field strength will expand into a spherical D2-D0-brane bound state. This situation would lift to M-theory as a spherical M2-brane carrying momentum in a direction orthogonal to an electric four-form. The latter is then essentially the "dual" giant graviton in the $AdS_4 \times S^7$ background.

The authors of ref. [1] provided a simple mechanical model [15] of an electric dipole, consisting of two separated charges held together by a simple linear restoring force and moving in a magnetic field, to provide some intuition for their expanding brane calculations. One might wonder if a similar mechanical model might describe the expansion of the dual branes induced by an electric fields. At first sight, the the answer appears to be no. If the dipole is oriented along the external electric field E, it is naturally extended to some equilibrium length L, but this length is unaffected by motion of the dipole transverse to the field. Actually this discussion would only apply for non-relativistic motions. If the transverse velocity v is relativistic, one finds there is a noticeable effect. In this case, to calculate the effect to the restoring force, we boost to the rest frame of the dipole. In this frame, however the electric field has increased to $E' = \gamma E$, and so the extension of the dipole is increased to L'. Upon boosting back to the original frame, the extension being transverse to the boost is unaffected and so the dipole appears to have increased in length. For small velocities, the variation is small, i.e. $\Delta L \propto v^2$, in contrast to the linear extension in a magnetic field, i.e. $\Delta L \propto v$. However, for large velocities $v \simeq c$, the extension is essentially proportional to the momentum, i.e. $\Delta L \propto |p|$. Of course, this is only a crude analogy for the physics of expanding branes. However, one observation is that reversing the dipole's velocity in the magnetic field reverses the orientation of the stretched dipole, but leaves it unchanged in the case of the electric field. In our brane analysis, this corresponds to the fact that the energy (2.29) for the "dual" giant gravitons is even in the angular momentum, but that in eq. (2.18) for the giant gravitons is not. This observation is also related to the appearance of expanding branes in some cases and expanding anti-branes in others. Perhaps the main lesson to draw from this model is the reminder that the various forces acting on the elements of the spherical branes transform in different ways under Lorentz boosts, which at least for certain configurations allows the expansion to take place when the branes are set in motion. From this point of view, this is reminiscent of the imbalance of forces arising in the scattering of supersymmetric solitons [16] or branes [17]. It would be interesting to investigate if these observations have any implications for theories with space-time noncommutativity [18].

The brane configurations all preserve one half of the 32 supersymmetries of the background $AdS_m \times S^n$ spacetime. The 16 supersymmetry transformations satisfying the "wrong" projection, i.e. $(\Gamma^{t\phi}-1)\epsilon_0 = 0$, would leave the background spacetime invariant but generate fermionic variations of the world-volume fields, which at the same time would leave the energy invariant. Of course, the equations of motion eliminate half of these to leave 8 fermionic zero-modes in each of the bosonic configurations studied here. These zero-modes are regarded as operators acting on a quantum space of states [19], which then build up for each bosonic configuration the full $2^8 = 256$ states of the supergraviton multiplet, as usual.

In general, working with the test-brane action will be problematic for the point-like configurations. Further, beyond the technical difficulties, we might expect that extra stringy or M-theoretic corrections to the dynamics, i.e. the test-brane description will breakdown. However, the supersymmetry of these configurations may be sufficient to protect the existence of these states in the full theory.

Note that in counting the candidate supergraviton states, we refer to a single point-like state. That is, we do not distinguish the collapsed $(m-2)$-brane from a collapsed $(n-2)$-brane. This is motivated by the Matrix theory description of M-theory in light-cone gauge [20]. There in principle one can represent various different branes with different geometries using non-commutative geometry, see e.g. [21]. However, when these geometries shrink to zero size, one is left with the same state, i.e. the same set of commuting matrices, independent of the original geometry. Even though we do not have a fully covariant formulation of M-theory, it seems natural to assume that it will still inherit this aspect of Matrix theory, and so the different collapsed branes in our calculations should correspond to the same point-like state.

In any event, it seems that we have an excess of potential states to describe the supergravitons carrying angular momentum. A natural question to ask is: are there even more expanded brane configurations with the same quantum numbers? Our attempts in that direction have failed to produced any additional configurations. The extra states beyond the original giant gravitons already present a problem, since these branes persist as stable configurations above the desired angular momentum bound of $P_\phi \leq N$. Certainly the nature of the supersymmetric states carrying angular momentum changes as P_ϕ increases past this bound, since the expanded branes of S^n can no longer contribute. From this point of view, the behavior is reminiscent of the “long strings” on AdS_3 [22]. In this case, macroscopic strings at infinity become relevant in describing the spectrum of states with scaling dimension above a certain bound.

However, if we are to interpret the stringy exclusion principle as saying there are no supersymmetric single particle states with angular momentum $P_\phi > N$, then the extra states seem to jeopardize the proposed explanation of the stringy exclusion principle in terms of expanding branes. We would like to suggest one possible way in which stringy exclusion principle might still be realized even though there are additional brane configurations beyond the giant graviton states of ref. [1]. First, one would expect that the three candidate states will mix quantum-mechanically. Certainly we have shown that there are instantons allowing for tunneling between either of the expanded brane configurations and the point-like states. It would be interesting to find instantons mediating tunneling directly between the giant gravitons and their “dual” cousins. However, it seems this would be beyond the scope of the test-brane framework used here. In any event, one might think that the three states mix and produce a unique ground state representing the true graviton.

To realize the exclusion principle, we further postulate that for angular momenta beyond the exclusion principle bound, $P_\phi > N$, there exists no supersymmetric ground state once the quantum mixing is taken into account. That is, the short supersymmetry multiplets associated with the point-like state and the “dual” giant graviton combine to give a massive long multiplet of states. Presumably the mass above the BPS bound is characterized by the instanton action, $\Delta\mathcal{H} \propto \exp(-\mathcal{S}^E_m)$. However, below the bound, $P_\phi < N$, there are three states and so the quantum mechanical mixing could only lift a combination of two short multiplets and must leave a unique supersymmetric ground state (see figure 6). The quantum wave function for this state would presumably still have support at all three of the candidate brane configurations.

This tentative realization of the stringy exclusion principle in terms of spontaneous symmetry breaking would be analogous to that in supersymmetric quantum mechanics [23] in generic models involving a single supersymmetric coordinate. In such models, there is a crucial difference between the cases where the bosonic potential has an even or odd number of supersymmetric minima. For an odd number there must be a supersymmetric ground state, while for an even number supersymmetry can be broken completely. In the current situation, we do have an odd number (three) of minima for $P_\phi \leq N$ (or more precisely, three minima for each set of quantum numbers in the 256-dimensional supergraviton multiplet). When we exceed the bound, $P_\phi > N$, one of the minima disappears, we are left with two minima, and this is the regime in which we might find spontaneous supersymmetry breaking. Of course, whether or not supersymmetry is actually broken in the present case will depend on the fermion zero-modes associated with the instanton. We leave determining the structure of these zero-modes as a calculation for future work.

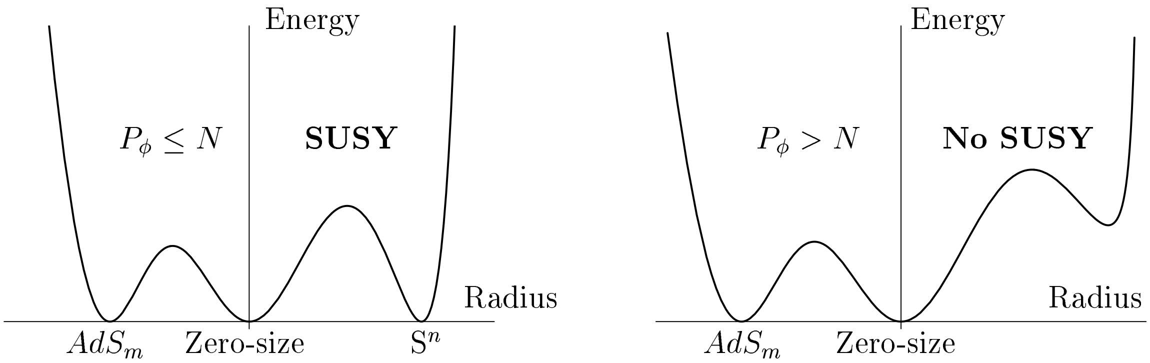

Figure 6: Part of the moduli space of giant gravitons. For $P_\phi \leq N$ there are three minima: the zero-size solution, an $(m-2)$-brane expanding into AdS_m, and an $(n-2)$-brane expanding into S^n. For $P_\phi > N$ the latter solution disappears — if supersymmetry is broken in this regime, then the stringy exclusion principle still holds.

It is interesting that by adjusting a parameter P_ϕ of the theory, one of the supersymmetric ground states disappears. This is unexpected behavior in that it would be impossible with a continuous parameter in a supersymmetric theory [23, 24]. However, in the present case, the angular momentum P_ϕ must be quantized to have integral values. This discreteness provides the loop-hole through which the short multiplet of supersymmetric states associated with the giant graviton is able to disappear. This behavior is reminiscent of Witten's result in three-dimensional $N = 1$ super-Yang-Mills theory with a Chern-Simons interaction [25]. There one finds that supersymmetry is spontaneously broken for a finite range of the level number, a parameter which can only take discrete values. Since the boundary conformal field theory for M-theory on AdS_4 is related to three-dimensional $N = 8$ super-Yang-Mills, it may be interesting to see if there is a concrete relation between Witten's results and the behavior of the giant gravitons in the $AdS_4 \times S^7$ background.

If one considers a Kaluza-Klein reduction to the AdS_m space, one has m-dimensional supergravity theory with an SO$(n + 1)$ gauge symmetry arising from the isometries of the internal S^n. From the point of view of this theory, the various brane configurations considered here are massive BPS states carrying a certain charge under a U(1) subgroup of the SO$(n+1)$. One might examine the supergravity to see if there are analogous charged black hole solutions [26]. However, one typically finds that these BPS configurations in AdS space correspond to naked singularities [27] rather than extreme RN black holes as in flat space. It seems then that the expansion of the giant gravitons is very closely related to stringy mechanism responsible for the removal of naked singularities by the enhançon found in ref. [28]. It would be interesting to investigate this analogy more closely.

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